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On the Variety of 'Genres' of Greek Mathematical Writing: Thinking about Mathematical Texts and Modes of Mathematical Discourse

Abstract: Scientists working today have a number of avenues open for the promulgation of their work. While electronic publishing of articles is now standard, new media, including podcasts and press conferences, are also used to publicize scientific research. Greco-Roman authors writing on scientific, mathematical and medical subjects also had a range of choices available to them as they selected the type of text to convey their ideas and information. Their choices included – but were not limited to – poetry, dialogue, lecture, question-and-answer text, letter, biography, recipe, epitome, encyclopedia, handbook, introduction and commentary. The consideration of the authorial choice of genre offers insights into how these writers regarded their own work, for example, in relation to the work of others. Furthermore, by choosing to write in a specific format, authors may have hoped to reach certain audiences; some texts are presumably more appropriate to students, others to specialists, still others to patrons or potential clients. And some types of texts have elements shaped by broader cultural convention rather than by the individual author. Given the range of options available to ancient writers on scientific, mathematical and medical topics, their choices of genre reflect authorial intention, including, for example, a desire to project a particular identity or image and/or to reach a special readership.

0 Introduction

Mathematicians and scientists working today have a number of avenues open for the promulgation of their work. While electronic publishing of articles is now standard, new media, including podcasts, are also used to publicize scientific research. These technologies enable the emergence of innovative forms of communication (for example, the 'sound-bite'), however the existence of a diverse range of options available for presenting scientific and mathematical material is not new. Surviving ancient Greco-Roman scientific, medical and mathematical texts display a surprising variety of forms, or genres, including, but not limited to, poetry, dialogue, lecture, question-and-answer text, letter, biography, recipe, epitome, encyclopedia and commentary. This empirically-derived short list suggests that ancient authors writing on scientific, mathematical and medical subjects had a number of options available to them as they sought to convey their ideas and information. To modern readers this is one of the most puzzling aspects of ancient scientific thought: the textual formats utilized for the exposition and dissemination of ideas. Furthermore, this area has not been the subject of much study. In my research I have examined the choice of medium used to convey the message, considering the implications, such as the effect of literary conventions associated with particular genres on the presentation of material by authors and subsequent reception by audiences. Here, I will concentrate on texts associated with mathematics.

A particular style of presentation, in a systematic format, is often seen by modern readers as the hallmark of Greek mathematics. As M. R. Cohen and I. E. Drabkin described it on the very first page of their *Source Book in Greek Science*, the characteristic mathematical text is the ideal "rigorously deductive proof, the method of developing a subject by a chain of theorems based on definitions, axioms, and postulates, and the constant striving for complete generality and abstraction".[1] Yet, upon further examination we see that the ideas and practices of ancient Greek mathematics were presented in a wide variety of types of texts, for the most part in prose formats, but occasionally in poems. Some of these texts were written by *mathēmatikoi*, men who presented themselves and were recognized by others as 'mathematicians'. But some of the texts that we would identify as 'mathematical' were written by non-specialists.[2]

There are many issues involved in the identification and description of different textual formats, types or genres, and there are also issues encountered in identifying and describing texts as 'mathematical'. In both cases, these are larger topics which cannot be dealt with fully, or resolved, here. Nevertheless, it is important to recognize that the categories being invoked are not entirely clear-cut and unproblematic. I will first consider some of the issues involved in the use of the term 'mathematical', and then turn to the challenges of defining genres of mathematical (and, more broadly, 'scientific') texts. While, increasingly, historians are considering the various forms and authorial intentions reflected in mathematical writings, such writings have not previously been discussed from the point of view of genre. I will consider first the Euclidean *Elements*, and then turn to a number of types of texts used by ancient authors to communicate about mathematics: the proposition, question-and-answer text, commentary, letter, and poem. While this is not an exhaustive list of the genres used by ancient Greek writers for mathematical discourse, here I can only touch briefly on some others – including *pragmateia* ('treatise'), *skholion* (lecture), *eisagōgē* (introduction) and *bios* (life) – simply to give a sense of the range of texts which should be considered.

1 Cohen & Drabkin 1958, 1.

2 See, for example, Cuomo 2001, 73–79 on this point.

1 The problem with 'mathematics'

When we look at an ancient Greek text, how do we know if it should be described as mathematics? What does it mean to use the terms 'mathematics' and 'mathematical' when reading and understanding ancient Greek texts? Is it the subject matter? The language? The vocabulary? The format or structure of the text? A style of argument? Is it through references made to the works of mathematicians? Is it through the use of certain techniques and tools, such as lettered diagrams? Historians of mathematics have not always agreed about the features that define a mathematical text. Several passages in Plato's dialogues, notably the *Meno* and the *Theaetetus*, are regarded as important in the history of mathematics. David Fowler highlighted the significance of certain passages in the dialogues, particularly the *Meno* (82a-85d), which he regarded as "our *first* direct, explicit, extended piece of evidence about Greek mathematics",[3] yet the Platonic dialogues are more usually treated primarily as philosophical texts.

Some ancient authors – primarily those identified as philosophers – wrote about the classification of different types of knowledge. Aristotle, in the *Metaphysics*, referred to the different types of knowledge (*epistēmē*), pointing to *mathēmatikē* as a distinct type of theoretical knowledge. Elsewhere Aristotle discussed the role of mathematics in its relation to other types of knowledge, including physics. Aristotle considered mathematics to be a type of theoretical knowledge, along with physics and metaphysics; he also outlined a system that classified some types of knowledge – including fields of mathematics, such as astronomy and optics – as subordinate to others.[4] Amongst those authors who wrote on such topics, not all agreed as to the classification and relationship of theoretical knowledge; so, for example, Ptolemy (2nd cent. AD) did not agree with Aristotle regarding the primacy of metaphysics, instead pointing to mathematics as the premier branch of philosophy.[5]

In addition to the ancient authors' classifications of knowledge (*epistēmē*), there are other distinctions that are evident in works written by, for example, practitioners, teachers and researchers. Pragmatically, such texts convey a sense of the field of endeavour in which they were produced and intended to be read by others. However, boundaries between specialisms of mathematical practice were not always as clear-cut as our modern descriptions of relevant texts and practices suggest. We must be mindful that any systematic and formal classification of knowledge and practice very likely only reflected in a limited way the more informal actors' categories of ancient authors and practitioners, and consumers (including

3 Fowler 1999, 7.
4 See, for example, Aristotle, *Anal. post.* 75b14–17; *Phys.* 194a7–8. See also McKirahan 1978; Lennox 1986.
5 Ptolemy, *Alm.* I 1. See also Taub 1993, 19–37. See also Sidoli 2004, 5–8.

readers, patrons, teachers and other users of texts). With our heightened awareness that the meaning of the descriptor ‘mathematical’ may not always be entirely clear, the difficulty of identifying different types or genres of mathematical texts becomes even more apparent.

2 The problem with ‘genre’

Ancient authors did not explicitly problematize their authorial choices as decisions about genre. There are few theoretical discussions of specific literary genres; in fact, Gian Biagio Conte and Glenn Most have noted that there was no theory of genre, as such, in antiquity. Most of the ancient authors are “more interested in classifying existing works than in understanding the mechanisms of literary production and reception and are directed to the needs of the school and the library, not to the [literary] critic’s”.[6] Those ancient Greek authors who did write on theoretical or taxonomical issues related to genre were not particularly concerned with prose, used for most scientific and mathematical texts. Poetry, drama and rhetoric were more gripping topics, and prose may have only been discussed under the rubric of rhetoric, encompassing speeches as well as historiography. Therefore, we cannot often turn to ancient discussions of scientific and mathematical texts to help us understand the significance of different forms of communication.

When the ancient term for a particular type of text is known, we have an idea of the ‘actors’ categories’ used to describe such formats and texts. Philip van der Eijk has pointed to the range of generic labels given to a number of stylistic formats found within the Hippocratic corpus.[7] However, in many cases we don’t have the author’s own label for the text (for example, either a title or a reference to the type of text), yet the form nevertheless seems clear; fortunately, in some cases we can see how ancient readers regarded the form of the text, through their references to it.

Modern scholars have sought to define ‘genre’, but the definitions are often hotly debated.[8] In 1974 Tzvetan Todorov, in a landmark article on “Literary Genres”, argued that a genre is always part of a system, “a certain horizon of expectation, i.e. a set of pre-existing rules which orient the reader’s understanding and allow him to receive and to appreciate the text”. Furthermore, genres “can only be

6 Conte & Most 1996, 631.

7 Van der Eijk 1997.

8 Today, the term ‘genre’ is increasingly used to classify non-literary and non-written forms of communication, including a type of painting, different types of music, and film, as well as speech acts. See Duff 2000, xiii. The term ‘genre fiction’ is used to refer to modern works of popular fiction that are regarded as highly standardized, for example historical romances, science fiction, and detective stories. Art historians use the term ‘genre painting’ to refer to a type of painting depicting ordinary activities, rather than historical or mythological subjects.

defined by their mutual relations" within the genre system. Todorov was particularly sensitive to historical issues, and emphasized that a genre must be redefined in each historical period, "in accordance with the other contemporary literary genres".[9] Focusing on ancient Greco-Roman texts, Conte and Most have also considered genre from the readers' point-of-view, emphasizing that genre is "not only a descriptive grid devised by philological research, but also a system of literary projection inscribed within the texts, serving to communicate certain expectations to readers and to guide their understanding."[10] They define 'genre' as referring to "a grouping of texts related within the system of literature by their sharing recognizably functionalized features of form and content".

Here I am largely concerned with written texts; in my treatment of mathematical writings I will use the following, suggested by David Duff, as my working definition of 'genre': "a recurring type or category of text, as defined by structural, thematic and/or functional criteria".[11] Following Duff's suggestions regarding structure, theme and function, and Conte and Most's emphasis on the functionalized features of form and content, it seems reasonable to begin a consideration of the genres of ancient scientific and mathematical texts by looking at form, content and function to help distinguish between different types of texts, or genres.

Having said that, it is worth remembering that literary specialists are themselves often wary of classifying texts. Wai Chee Dimock, in a special journal issue of the *Publications of the Modern Language Association of America* dedicated to "Remapping Genre", opened her introduction by asking "What exactly are genres? Are they a classifying system matching the phenomenal world of objects, a sorting principle . . . ? Or are they less than that, a taxonomy that never fully taxonomizes, labels that never quite keep things straight?". She answered by arguing that no genre "is a closed book, none an exhaustive blueprint. . . . Far from being a neat catalog of what exists and what is to come, genres are a vexed attempt to deal with material that might or might not fit into that catalog. They are empirical rather than logical".[12] Dimock's cautions regarding tidy categorization are apt; in considering genres of ancient Greek mathematical written texts, I have purposely adopted a non-theoretized methodology, choosing to pursue what may be regarded as a 'from the ground up', largely empirical, approach which proceeds from the texts themselves. My treatment begins with a close reading of the text, and I intend description of texts to support my argument. When possible, I aim to be mindful of actors' categories, as well as the broader contexts in which the texts were produced, circulated and read. Genres reflect expectations, as well as conventions.

9 Todorov 1974, 958. There is a vast and voluminous scholarship on the question of genre, too extensive to be referred to in any comprehensive way here.

10 Conte & Most 1996, 631. See also Conte 1994, 105–128. Depew & Obbink 2000, 1–14, provides a useful overview of some of the issues surrounding genres of Greek and Roman literature.

11 Duff 2000, xiii.

12 Dimock 2007, 1377–1378.

The categorization of genres is not always clear-cut; some texts also combine features of multiple genres, forming a sort of hybrid text. This evidence of 'hybrid' texts suggests that ancient authors and their readers may have had a relatively high tolerance for variation.[13] Questions of normativity, regarding the features of a specific genre, are an historical problem, as Todorov suggested. Ancient authors and readers had different expectations than ours, and likely had a different degree of adaptability and flexibility in composing and encountering mathematical texts than do their modern counterparts.

The taxonomy I offer has been arrived at by empirical means, attempting to consider, particularly, form and function. But even these distinctions are not always clear-cut: a particular text may have sections which reflect a number of genres. Similarly, as we have seen with Plato's dialogues, an individual text may have had more than one function, for example a teaching text may have been used to attract students, not simply as a pedagogical tool. Here I concentrate on texts whose content is, broadly speaking, 'mathematical'; in many cases I am guided by the ancient authors themselves indicating that they are writing about mathematics, or about the work of a particular mathematician.

3 Authorial choices

As already noted, ancient Greek authors had a wide range of options in the type of text they used for communicating their ideas and information; some of these were borrowed from existing forms, others they created for themselves. To some extent, textual formats represent choices which reflected authorial intention, but the extent to which the use of a particular format reflects an intentional choice made by an author (or editor) is open to debate, and not always clear to us. Nevertheless, in some instances there are clear indications that the author deliberately exercised choice; for instance, the "Letter to King Ptolemy", one of the texts discussed in some detail below, incorporates a number of types of text (including the proof and the epigram) into the epistolary format.

Some choices made by ancient authors – for example, the decision to write in hexameter verse – could immediately place the text within the broader traditions of epic and didactic poetry.[14] In other instances, the relative cultural weight of the decision to employ a particular type of text is not immediately clear to us. For other, less obviously literary, formats, it is not always clear what these choices implied to their authors and intended readers. For example, as Todd Curtis has

13 Netz 2009, 129–136 discusses 'hybrid' treatises, from a different perspective.

14 The extent to which authors have a 'choice' or make a 'decision' to write in, say, hexameter, cannot be addressed here; I do recognize that broader cultural norms and constraints may operate, limiting 'choice'.

shown, Galen's decision to present some of his ideas on the medical study of the pulse in the style of an introductory text, offered to beginning students, involved a complex interplay with his other treatments of the same topic intended for more advanced readers/practitioners.[15]

A focus on ideas alone occludes other important information conveyed by authors through their adoption of particular voices and genres. Classicists have traditionally made a strong break between literary and non-literary texts. A certain number of important technical and scientific texts have received a great deal of attention as literature, particularly the works of Lucretius, Vergil and Aratus. Furthermore, the rhetoric of scientific and technical texts has been recently addressed within the context of a wider move to explore the centrality of rhetoric to ancient Greco-Roman literature and culture. However, when classicists and historians of science, mathematics and medicine consider such texts, the tendency has been – generally – to ignore the genre of communication, concentrating instead on the content and ideas. There has been little work done to improve our understanding of the dynamics of authorial choice and reader expectations established by a scientific text's genre.

Ancient Greek mathematical texts have often been regarded as being characterized by their impersonal style. Professional scientific writing in the contemporary world generally avoids the use of the first person and adopts an impersonal or depersonalized style. Yet, in antiquity, the creation of a distinctive voice or persona was often central to the process of establishing one's authority as a scientific or medical author. The question of authorial voice is in some cases key to understanding these texts, even when the author is unidentified or unknown to us. Strategies of self-presentation have been considered by a number of scholars working on technical texts, not only in the ancient period.[16] Thorsten Fögen has considered the Elder Pliny's strategies of self-presentation through which he aims to come across as scholarly and authoritative, in some cases supporting the views of his predecessors, whilst in other instances distancing himself from them.[17] In certain mathematical texts, the creation of an impersonal, disembodied voice distinguished those texts. In contrast, as Vivian Nutton has noted, Galen frequently adopts self-referential personal forms, compared with other writers (including Rufus of Ephesus (*ca* 70–100 AD) and Aretaeus of Cappadocia (150–190 AD?) who tend to use more neutral language.[18]

Genre may also be used to target certain audiences; some texts are more appropriate to students, others to specialists, still others to patrons, clients, etc. Remembering that genres were sometimes developed and used for specific areas of or

15 Curtis 2009.
16 See, for example, the contributions in Biagioli & Galison 2003, on scientific authorship.
17 Fögen, in this volume.
18 Nutton 2009, 59.

approaches to knowledge (for example, the encyclopedia, developed by Pliny the Elder to display a breadth of knowledge),[19] my focus here is on formats and genres used to communicate about mathematics, noting that others have written about the diversity of genres of medical writings, for example, in the Hippocratic Corpus itself.[20]

4 Genres of mathematical writing

Using the term 'mathematical' as a label might suggest that there is unanimity in understanding this descriptor.[21] In some cases, it is not clear whether a text should be labeled as 'mathematical', or what, precisely, that label might entail. There is also the danger of applying such terms ahistorically, by suggesting that the modern usages map onto those of ancient authors and practitioners. Modern readers do not always agree as to what characterizes a mathematical text; in fact, ancient authors who wrote about mathematics did not always agree in its definition either.

Furthermore, there is no precise agreement as to what distinguishes a mathematical text from one that is not mathematical. (For example, Nathan Sidoli, in his treatment of Ptolemy's mathematical discourse, "omits passages which may be about mathematics but do not form part of the mathematical argument"; he refers to such material, which may include introductory material such as definitions and first principles, as "discussion".)[22] Reviel Netz has emphasized the use of technical language and lettered diagrams as key features of Greek mathematical texts;[23] Sidoli has argued that "the basic elements of Greek mathematical exposition are words, numbers and diagrams".[24]

As work by Serafina Cuomo and others has shown us, in this volume and elsewhere, ancient mathematical practices can be seen as a spectrum. The texts associated with these different practices are, likewise, somewhat different in form, with sophisticated treatises such as Archimedes' *Method* at one end, and texts such as multiplication tables and account inscriptions at the other.[25] Perhaps

19 See, for example, Doody 2010; Murphy 2004.
20 Van der Eijk 1997.
21 I am grateful to Bernard Vitrac for corresponding with me about these issues.
22 Sidoli 2004, 9 restricted his discussion of mathematical prose to include only those portions of texts that "do" math, rather than speak about math.
23 See Netz 1999a, 12–67.
24 Sidoli 2004, 8.
25 See Cuomo, in this volume. Recently, there has been a shift in attention to texts and sources which reflects wider mathematical practices. Questions such as the way in which a history of numeracy may differ from a history of mathematics have been posed. On such questions in the Greek context, see Cuomo 2001; in the Babylonian context, Robson 2009; in the Egyptian, Imhausen 2003.

unsurprisingly, scholarship has often concentrated on authors at the high end of the spectrum. Markus Asper has suggested that there were two cultures of people engaged in two rather distinctive types of mathematics in Greek antiquity: practitioners, on the one hand, and a more elite group of theoreticians, on the other.[26] But there were individuals, notably, for example, Archimedes (third cent. BC) and Hero (first cent. AD), who crossed whatever boundaries might have existed between these two groups.

Cuomo has noted that mathematics was associated "not only with a certain subject-matter (numbers, geometrical figures), but also with a certain style".[27] Aristotle had earlier recognized this, noting that for some styles of argument and presentation audiences have clear expectations: "Some people do not listen to a speaker unless he speaks mathematically, others unless he gives instances, while others expect him to cite a poet as witness".[28]

Whether there was a distinct genre of 'mathematical text' in antiquity is a question to be considered. Those authors writing about mathematical topics used a variety of formats, including some that look similar to question-and-answer texts; other texts are deliberately cast as letters, in some cases addressed to specific individuals, including patrons.[29] To give an idea of the sort of variety that exists, we might include the following as types of texts relevant for ancient Greek mathematics: proposition, letter, problem text, dialogue, poem, commentary, treatise, lecture, introductory text, narrative, and biography. However, this is not intended as a complete list of all genres or formats used for communicating mathematical ideas and methods by ancient Greek authors; others might include the handbook. Certainly, some of those listed have particular relevance for mathematical texts.[30]

It must be emphasized that these labels cannot be taken to always represent strict divisions between formats, or a hard and fast taxonomy; some dialogues, for example, Plato's *Timaeus*, which has sections which are usually regarded as mathematically-informed, reads almost like a monologue, or lecture. Furthermore, some texts may contain elements of a number of genres and there are some overlapping categories. So, for example, some 'teaching texts' are written as poems; in considering prose writings, there seem to be various types of texts, but it is sometimes difficult to know how to distinguish them. Even within a particular genre of text, such as the commentary, there may be a number of other genres of writing contained within that larger text.

There are some types of texts which are particularly associated with the writings of ancient Greek mathematicians. Many examples of letters written by mathe-

26 Asper 2009.
27 Cuomo 2001, 32.
28 Aristotle, *Metaph.* 995a5–7 (1572).
29 On Eratosthenes' "Letter to King Ptolemy", see Taub 2008b.
30 Netz 1999b, 282 has noted that there has not been a great deal of interest in stylistic features of ancient Greek mathematics, with a preference generally for concentrating on contents and logical forms.

maticians, including Archimedes, survive; sometimes these serve as an introduction to a mathematical text which is itself presented in a different format (such as a proposition). Introductions, even those in the form of a letter, have been regarded as somewhat ancillary to the main text; the literary theorist Gérard Genette described introductions (and other 'boundary' objects used in published work) as "paratexts". He did, however, recognize that paratexts convey important messages, and even serve to mediate and shape the reading of the main text; they may well adhere to certain conventions (for example, of address) and rhetorical forms. Genette was concerned with modern printed works, but the concept of "paratext" has also been applied, by Asper, to the letter-as-introduction used by ancient Greek mathematicians.[31] As an example of such a paratext and text, Archimedes begins a letter to Dositheus: "Greetings. Earlier, I have sent you some of what we had already investigated then, writing it with a proof". This serves as an introduction to the text *On the Sphere and the Cylinder*, most of which is presented in the form of propositions and proofs.[32]

5 The archetypal mathematical text: The *Elements*

For many readers, the term 'mathematics' brings to mind a distinctive type of text, one that exhibits a particular linguistic style and form of presentation. Many ancient Greek mathematical texts have their own character, which will be familiar particularly to students of geometry. So, for example, as has already been noted, the use of technical, formulaic language and lettered diagrams are sometimes regarded as key features of Greek mathematical texts. The *Elements* of Euclid, which relies on such features in abundance, often serves as the archetypal ancient Greek mathematical text. Historically, the *Elements* has loomed large, and shaped expectations of what mathematical texts, and indeed, particularly in later periods, what scientific texts should look like. (Interest in the formal qualities of mathematical texts is still important to mathematicians today; with this in mind, in October 2004 the Royal Society held a special two-day discussion in London about mathematical proof.[33])

It is almost a truism that many of the 'high-end' mathematical texts, such as the *Elements*, are associated with what may be regarded as the distinctive voice of a particular author, a particular individual, such as Archimedes or Ptolemy.[34] Having said that, the *Elements* is now thought to be the work of compilation, rather

31 Asper 2009, 118; see also Genette 1997, 1–15 *et passim*.
32 See Netz 2004, 31, *et passim*.
33 Netz (in this volume) explores the idea of genre in mathematical texts from a different perspective, i.e., that of authorial presence.
34 See Netz 2002 and 2009 on Archimedes' style.

than that of a single author; how that might be reflected in 'authorial' voice is a not entirely clear.[35] Historians of mathematics believe that the *Elements* was, in part, synthesized and systematically presented by Euclid in *ca* 300 BC; the thirteen 'books' cover a variety of topics in a range of mathematical and literary styles.[36] David Fowler has emphasized that Euclid should be understood as "the compiler, not the author, of the work: he is believed to have taken source works by other mathematicians and edited them, adapting and rearranging the material, perhaps even inserting new material of his own, to make the complete treatises".[37]

While the style of presentation within the *Elements* is not completely uniform, a particular format is characteristic: that of the proposition and proof. Sidoli has described the proposition as the "basic unit of mathematical prose".[38] In addition to multiple examples of propositions (and proofs or solutions), the *Elements* has certain other important features, in particular the statement at the very beginning of the text, of what may be regarded as 'the essential preliminary matter', classified under the headings *Definitions* (*horoi), Postulates* (*aitēmata*) and *Common Notions* (*koinai ennoiai*).[39] The format of the proposition is often seen as not only characteristic of but, indeed, definitive of mathematical discourse.

6 Proposition

Modern terminology to describe and distinguish various elements of formal mathematical texts is not universally agreed; we have evidence too that in antiquity

35 Diogenes Laertius and Pliny the Elder are also sometimes described as 'compilers'; they each make reference to their numerous sources.

36 Books 1–4 are concerned with plane geometry, book 5 treats the theory of proportions, and book 6 deals with the similarity of plane figures. Books 7–9 are concerned with number theory, book 10 with commensurability and incommensurability, books 11–12 treat three-dimensional geometric objects, and book 13 the construction of the five regular solids. Later non-Euclidian additions include book 14, which may be due to Hypsicles of Alexandria (ca 200 BC), and book 15, which may be at least partly the work of a sixth-century pupil of Isidorus of Miletus. See also Mueller 2008. On the history of early modern editions and translations of the texts, see the Brown University Library online exhibition *From Euclid to Newton: An Exhibition in Honor of the 1999 Conference of the Mathematical Association of America*.

37 Fowler 1999, 205. On Euclid's work as a compiler and editor, see Knorr 1975, 303–312. On the editions of Euclid, including the recension by Theon of Alexandria, see Heath 1921a, 360 f. Cf. to Eutocius' role as a commentator on and compiler of an anthology of solutions to the Delian problem, Heath 1921b, 540 f.

38 Sidoli 2004, 8.

39 Not all of the definitions are used in the *Elements*; Heath believed that some may have been included out of a respect for tradition. Certainly, the influence of Aristotle, and possibly Plato too, is evident in the setting out of the preliminary terminology. Heath 1921a, 373. As noted above, Sidoli regards definitions, etc., as non-mathematical "discussion". Sidoli 2004, 9.

there was felt the need to discuss difficulties encountered in the naming of parts of such texts. A mathematical proposition is a formal statement of a theorem (which is to be demonstrated) or a problem (which is to be solved).[40] In his *Commentary on Euclid's Elements*, Proclus (412–485 AD) discusses the distinctions made by different authorities between theorems and problems, making it clear that not every author used these terms in the same way.[41]

The terminology of propositions was considered important enough that several ancient authors wrote extensively on the subject.[42] Pappus, a fourth-century mathematical author, also discussed these terms, in the preface to book 3 of his *Mathematical Collection*, but it is not entirely clear when particular items of terminology were first adopted.[43] Other technical terms, such as "lemma" (something assumed), "porism" (some result incidentally revealed in the course of the demonstration of the main proposition under discussion), "analysis" and "synthesis" are also discussed by Proclus (*In Eucl.* 211–213; 255–266); Pappus discusses analysis and synthesis in the *Mathematical Collection*, book 7.[44] The structure and varieties of propositions, as well as their relationship to mathematics more generally, have been the subject of study since antiquity, as well as the technical terminology used in mathematical texts.

The crucial form of presentation within the *Elements* is the proposition. Keeping in mind the definition of 'genre', as a recurring type or category of text, as defined by structural, thematic and/or functional criteria, ancient authors, such as Proclus and Pappus, as well as modern scholars, including Heath (in his work on Archimedes and Apollonius, as well as on Euclid) and Netz, have addressed the characteristics of the proposition in a way which suggests that it might be regarded as a genre in itself.[45]

Technical terminology was clearly a subject of discussion itself in antiquity. As part of his rather lengthy discussion of the first proposition, Proclus briefly lists and explains the formal divisions, and their functions, contained therein:[46]

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40 Cf. Sidoli 2004, 8.

41 Proclus, *In Eucl.* 77.7–81.2. See also Mueller 1981, 11; Knorr 1986, 348–360; Netz 1999b, 288; Sidoli 2004, 8–9.

42 Cf. Heath 1921b, 533f.

43 See Netz 1999b on this latter point.

44 Cf. Heath 1921b, 533. On definitions of lemma, porism, etc., see Heath 1921a, 372 f. All references to Proclus are to the Friedlein edition of the commentary of the first book of Euclid, unless otherwise noted. For a translation of the passage from Pappus (book 7) on the Definition of Analysis and Synthesis, see Heath 1921b, 400f. On Pappus, see Jones 1986, particularly 1–3, 66–74; Cuomo 2000.

45 Heath included separate sections on terminology in his translations of Archimedes (1912, clv–clxxxvi) and Apollonius (1896, clvii–clxx). In his translation of Euclid, Heath (1925/1956) included sections dealing with terminology in his first chapter, treating "Theorems and Problems", "The Formal Divisions of a Proposition", and "Other Technical Terms". See also Netz 1999b.

46 Proclus, *In Eucl.* 203.1–15, Morrow transl. 1970/1992, 159, which I have adopted, with a few emendations; the fuller discussion of the parts of the proposition occupies 203.1–210.6. Proclus'

- The enunciation (*protasis*) "states what is given and what is being sought from it, for a perfect enunciation consists of both these parts". The claim of the proposition is stated in general terms; the *protasis* is equivalent to a conditional statement that *if x, then y*.[47]
- The specification (or setting-out; *ekthesis*) "takes separately what is given and prepares it in advance for use in the investigation". As Heath notes, the *ekthesis* "states the particular *data*", for example, "a given straight line *AB*, two given triangles *ABC*, *DEF*, and the like, generally shown in a figure and constituting that upon which the proposition is to operate".[48]
- The definition or specification (*diorismos*) takes "the thing that is sought and makes clear precisely what it is". It restates what is required to be done or to prove in terms of the particular data already stated; a statement of the conditions of possibility may also be contained in the *diorismos*.[49]
- The construction (*kataskeuē*) "adds what is lacking in the given for finding what is sought", including any additions to a figure by way of construction that are necessary to enable the proof to proceed.
- The proof (*apodeixis*) itself "draws the proposed inference by reasoning knowledgeably (or, in a manner capable of knowledge, or scientifically, *epistēmonikōs*) from the propositions that have been admitted", to prove the particular claim.[50]
- The conclusion (*sumperasma*) "reverts to the enunciation, confirming what has been proved" or accomplished. As Heath points out, "the conclusion can … be stated in as general terms as the enunciation, since it does not depend on the particular figure drawn; that figure is only an illustration, a type of the *class* of figure, and it is legitimate therefore, in stating the conclusion, to pass from the particular to the general".[51]

The first proposition presented in the *Elements* serves as an example, for Proclus himself takes his audience through it in detail, examining the formal structure: "Let us view the things that have been said by applying them to this our first problem. Clearly it is a problem, for it bids us devise a way of constructing an equilateral triangle".[52]

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formal division of the proposition is discussed by Heath 1921a, 370 f., Heath 1925/1956, 129–131, and Netz 1999b.

47 Netz 2004, 6; Netz 1999b. There is no reference to a diagram in the enunciation.

48 Heath 1921a, 370; cf. Netz 2004, 6.

49 Netz 2004, 6, regards the *diorismos* as an "exhortation by the author to himself". Cf. Heath 1921a, 371. See also Thomas 1939, 394–397 and his discussion of the *diorismos* in the Meno, and Knorr 1986, 73–74.

50 Proclus, *In Eucl.* 203.12–13: ἡ δὲ ἀπόδειξις ἐπιστημονικῶς ἀπὸ τῶν ὁμολογηθέντων συνάγει τὸ προκείμενον.

51 Heath 1921a, 370.

52 Proclus, *In Eucl.* 208–210, Morrow 1970/1992, 162–164. Netz 1999b, 284 noted that it is not the ideal example.

Its format is explicated by Proclus as follows (I have placed the corresponding passage from the *Elements* in brackets following):[53]

– The *protasis* (enunciation, which in this case he explains consists of both "what is given and what is being sought"): "*If there is a finite straight line, it is possible to construct an equilateral triangle on it*". (On a given finite straight line to construct an equilateral triangle.)
– The *ekthesis* (the exposition, or setting-out): "*Let this be the given finite straight line*". (Let *AB* be the given finite straight line.)
– The *diorismos* (definition or specification): "*It is required to construct an equilateral triangle on the designated finite straight line*". (Thus it is required to construct an equilateral triangle on the straight line *AB*.)
– The construction (*kataskeuē*), which includes any additions to the original figure by way of construction that are necessary to enable the proof to proceed: "Let a circle be described with center at one extremity of the line and the remainder of the line as distance; again let a circle be described with the other extremity as centre and the same distance as before; and then from the point of intersection of the circles let straight lines be joined to the two extremities of the given straight line".[54] (With centre A and distance AB let the circle BCD be described; again, with centre B and distance BA let the circle ACE be described; and from the point C, in which the circles cut one another, to the points A,B let the straight lines CA, CB be joined.)
– Next comes the proof itself, in which the particular claim is proven: "Since one of the two points on the given straight line is the center of the circle enclosing it, the line drawn to the point of intersection is equal to the given straight line. For the same reason, since the other point on the given straight line is itself the center of the circle enclosing it, the line drawn from it to the point of intersection is equal to the given straight line ... Each of these lines is therefore equal to the same line; and things equal to the same thing are equal to each other ... The three lines therefore are equal, and an equilateral triangle [ABC] has been constructed on this given straight line". (The elisions here represent the omission of Proclus' comments on the proof.)
  (Now, since the point A is the center of the circle CDB, AC is equal to AB. Again, since the point B is the center of the circle CAE, BC is equal to BA. But CA was also proved equal to AB; therefore each of the straight lines CA, CB is equal to AB. And things which are equal to the same thing are also equal to one another; therefore CA is also equal to CB. There the three straight lines CA, AB, BC are equal to one another. Therefore the triangle ABC is equilateral; and it has been constructed on the given finite straight line AB.)

53 Euclid, *Elem.*, transl. Heath (1925/1956), I 241–242.
54 Friedlein presents the text in a corrupt state; Morrow's transl. follows Francisco Barocius' 1560 text here (*Procli Diadochi Lycii in Primum Euclidis Elementorum Commentariorum Libri IV a Francisco Barocio Patritio Veneto Editi*, Padua, 1560).

- Proclus then states the general conclusion, the *sumperasma*: "An equilateral triangle has therefore been constructed upon the given straight line". In the *Elements*, there is no formal conclusion (*sumperasma*) for the first proposition, restating in general terms what was to be proved or done, but simply an assertion that what was required to be done was accomplished: "[Being] what it was required to do".

  Proclus notes that Euclid adds: "This is what it was required to do", thus showing that this is the conclusion of a problem; Proclus explains that in the case of a theorem, Euclid states: "This is what was to be demonstrated" (the equivalent of our Q.E.D.).[55]

In actual fact, according to Proclus, not all propositions have all of the formal divisions listed above, even though the enunciation, proof and conclusion are (once again, according to Proclus) always found.[56] So in many propositions no construction is needed, as the figure given is itself sufficient for the proof; Proclus noted that in the problem "to construct an isosceles triangle with each of the base angles double the other angle" there is neither a setting-out nor a definition.[57] (In addition to the lack of uniformity with regard to the form of propositions within the *Elements*, and subsequent mathematical texts, there are also different styles of proof; while this is important from the standpoint of the mathematical argument being made, it is perhaps less important in defining a genre.[58])

It is clear even from Proclus' *Commentary* on the *Elements* that the formal character of the geometrical proposition was an object of study in itself; certain formal features could be considered as characteristic, and expected, serving specific functions.[59] Other writers also concerned themselves with explaining features of mathematical texts. As was mentioned earlier, the fourth-century AD mathematical author and commentator, Pappus of Alexandria, is credited with a *Mathematical Collection* in eight books, in Gerald Toomer's view a compilation probably made after his death of originally separate works on different mathematical topics; not all of the *Collection* survives.[60] As was Proclus, it is clear from Pappus' discussion at a number of places in this work that he was concerned with the form of mathematical texts; so, for example, in the preface to book 3, he discussed the character

55 Proclus, *In Eucl*. 210, Morrow 1970/1992, 164.

56 Proclus, *In Eucl*. 203, Morrow 1970/1992, 159. See also Heath 1921a, 371.

57 This problem is found in *Elements* IV 10; cf. Proclus, *In Eucl*. 204, Morrow 1970/1992, 159; Cf. Heath 1921a, 371.

58 For example, on the proof by analysis, see Heath 1921a, 371 f. Heath 1925/1956, 136–137 discusses the use terms by Aristotle, Pappus and Proclus. The *Elements* has many examples of *reductio ad absurdum*; the first being in book 1, Proposition 6. See Heath 1925/1956, 255–256.

59 Netz (1999b) has argued that Proclus has developed his own terminology and exegesis in his *Commentary*; from my standpoint, whether or not the terminology and breakdown of the proposition pre-date Proclus is immaterial to my argument.

60 Toomer 1996b, 1109.

of *problems* and *theorems*.[61] Proclus himself had provided information about the (in some cases, contrasting) views of some of his predecessors, including Carpus and Geminus.[62]

While the formal structures of the proposition were adopted by other mathematical authors – including Archimedes in various of his writings, Apollonius (in the *Conics*), and Eratosthenes, in his report of the duplication of the cube – in some cases they deviated from the exact, idealized, format of the proposition as described by Proclus. (It is important to recognize that the invention of this schematic format for the proposition may have been due to Proclus himself.) While, to some extent, the *Elements* served as an exemplar text, as Netz has noted, Archimedes "had many variations on the Euclidean structure. General conclusions are avoided, and construction, setting-out, and proof are often intermingled".[63] Within the expected formal structure of the geometrical proposition, there was a degree of variability, even license, as to what specific features might or might not be included by individual authors and editors.

The proposition can be considered to be a genre of mathematical text, but it is not only used in mathematical texts; logical texts also employ propositions and proofs, though the specific characteristics of these vary. Generality is one of the key features of the geometrical proposition, a feature also shared with logical propositions, as Aristotle explained in the *Prior Analytics*.[64] Further, the proof is not a format confined to mathematics; indeed, the question of the relationship between logical and geometrical proofs has been investigated by historians, and there is a considerable literature on this topic.[65] Aristotle discussed the structure of geometrical proofs in his *Prior Analytics* I 24.[66] The ambition to provide a generalized explanation in the form of a proposition and proof is emphasized by the choice made by many Greek authors to communicate via a text employing general terms; this characteristic generality helps to explain why this type of text – the proposition and proof – has been regarded by some as the ideal format for mathematical and scientific explanation. However, Greek mathematics does not require (in a logical sense) this explicit generality; Euclid without *protaseis* and conclusions would still be mathematics.[67]

61 In addition to his interest in mathematical texts, Pappus was also concerned with questions relating to the practice of mathematics more generally. For example, Pappus (*Coll.* V, preface 1–3) contrasts the mathematical "practice" of bees to the mathematics accomplished by humans; see also Cuomo 2000, 57–90.

62 Proclus, *In Eucl.* 241–244 (Morrow 1970/1992, 188–191). See also Knorr 1986, 348–360.

63 Netz 2004, 120.

64 Aristotle, *Anal. pr.* 41b6–26.

65 See, for example, Frede 1974; Mueller 1981, 11–15. See also, e.g., Smith 1989, 111–112; Fowler 1999, 388–390.

66 See also Smith 1989, 144.

67 I thank Ian Mueller for this suggestion (personal communication); it might even be argued that Euclid would be clearer without *protaseis* and conclusions.

7 Question-and-answer (problem) texts

The focus of attention on a proposition to be demonstrated (if a theorem) or solved (as a problem) is a feature shared by many texts concerned with mathematics.[68] As was hinted above, the terminology used was not always precisely delineated, and we find problems (*problēmata*) presented in a number of types of texts, in some cases, with solutions, in others not.

Recalling Fowler's suggestion that the *Meno* may represent "our *first* direct, explicit, extended piece of evidence about Greek mathematics",[69] the genre of dialogue, particularly in the form of the Socratic model devised by Plato, can be understood more generally as being extremely well suited to the presentation of problems. However, very few of the 'problems' presented in the Socratic dialogues are concerned with mathematics; rather, Plato was concerned with philosophical issues. Plutarch, one of the few ancient authors to compose a dialogue concerned with scientific issues, also presented some mathematics in his dialogue *On the face on the moon*. But of all the interlocutors named in the dialogue, it is only the one described as a mathematician, Theon, who never himself speaks; Plutarch presents the mathematician as a silent participant in the discussion of the problems posed.[70]

Aristotle, while he was at Plato's Academy, is understood to have compiled notes on various "difficulties" that intrigued him; this collection of problems was available to members of his own school, the Lyceum.[71] Over time, a number of Peripatetic philosophers added to the collection. While the text known as the *Problems* in the Aristotelian corpus has the stamp of his school, the work was apparently compiled over a period of time and may not have reached its present form before the fifth century AD; in other words, it may not be the work of one individual, but many.[72]

Other authors and/or compilers also produced collections of 'problems' as texts; some *problēmata* texts deal with nature, some with literature. Question-and-

68 Knorr 1986, 349 has pointed out that "from the purely formal viewpoint the distinction between problems and theorems is largely artificial. One can easily recast any problem as a theorem, merely by incorporating into the *protasis* of the theorem all the details of the construction of the problem".

69 Fowler 1999, 7.

70 The intriguing nature of Theon's silence cannot be addressed here. But, see Netz (in this volume) on the silence of mathematicians.

71 Louis 1991, xxiii–xxxv; cf. Inwood 1992. The compiling of a collection of difficulties and problems resonates with other aspects of Aristotle's activities, including the forming of a collection of constitutions, as well as his suggestions for taking reading notes and making lists of opinions.

72 Scholars tend to agree that the author of the so-called Pseudo-Aristotelian *Problems* (*Problēmata*) is not Aristotle, although Aristotle is known to have written a book of problems. Some of the material included in the Pseudo-Aristotelian *Problems* seems actually to have its source in the work of Aristotle; several ancient authors (including Plutarch and Cicero) described portions of the *Problems* as Aristotelian. See Hett 1936, vii. Cf. also Louis 1991.

answer texts follow a basic pattern in which a question is posed and an answer is provided. The answers may range from rather brief (a few lines) to somewhat lengthy (the equivalent of several pages). Questions are not necessarily related to one another, although in some cases questions on similar topics are grouped together.[73] There is an argument for suggesting that certain logical and mathematical texts (particularly propositions) can be understood as related to these *problēmata* or question-and-answer texts; in certain geometrical texts, for example Euclid's *Elements*, problems are presented and solved; Hellmut Flashar has noted that geometrical problems imply a task to be completed.[74]

The pseudo-Aristotelian *Problems* is composed of thirty-eight books, covering a wide range of subjects, from problems connected with medicine (book 1) to problems concerned with mathematical theory (book 15), and questions about shrubs and plants (book 20).[75] The following (question 10) is an example of the sort of 'problem' presented in book 15, as one of the questions concerning mathematics:[76]

> Why are the shadows thrown by the moon longer than those thrown by the sun, though both are thrown by the same perpendicular object? Is it because the sun is higher than the moon, and so the ray from the higher point must fall within that from the lower point? Let AD be the gnomon, B the moon, and C the sun. The ray from the moon is BF, so that the shadow will be DF; but the ray from the sun is CE, and its shadow therefore will necessarily be less, viz. DE.[77]

Here, a question arising from observation – the length of shadows – is answered by means of a geometrical demonstration. However, it is not clear that this problem is about mathematical theory; rather, the question concerns the shadows cast by the sun and moon. Here, a geometrical demonstration is used to present an argument about phenomena.

Another problem text, the pseudo-Aristotelian *Mechanical Problems* (or *Mechanica*) is thought to be the earliest surviving text on the mechanics, and

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73 Hine 1981, 27–29. See also Cherniss 1976/2000, 2–5, in which he discusses the *zētēmata* literature, which posed questions concerned with the meaning of a passage in a text (traditionally in Homer, but also applied to other texts as well). Collections of questions focusing on nature include the pseudo-Aristotelian *Problems* and Plutarch's *Natural Questions*.

74 Flashar 1975, 298.

75 On the *Problems* see Flashar 1975 and Sharples 2006, as well as the other articles in de Leemans & Goyens 2006.

76 One of the reasons I chose this question as an example is because it refers to another piece of writing – in this case a drawing or diagram. I have decided against providing a diagram here as, to my knowledge, no ancient version survives. While the text suggests that it was accompanied by a diagram, it may have been left for readers or students to construct themselves. However, there are similar references to such visual aids in works by Aristotle, for example the *Meteorology*; such diagrams may have been included in a text, or displayed to an audience during a lecture. Taub 2003, 103–115; Netz 1999a, 37. See also Sider 2005, 15–19 on diagrams in ancient texts.

77 Ps.-Aristotle, *Probl.* 912b4–10, transl. Forster in Barnes' ed., 2.1419, question 10.

includes thirty-five problems. Like the pseudo-Aristotelian *Problems*, the problems posed in the *Mechanical Problems* (*MP*) are presented as questions to which answers are (usually) given; for the most part, a geometrical proof is offered. Sylvia Berryman describes the *MP* as a "treatise"; W.S. Hett has suggested that the *Problems* originally may have been a series of lecture notebooks organized by subject area, to which new problems and answers were continually being added.[78] The problems contained in the *MP* may have also had a pedagogical function. Certainly, the posing of problems for solution was a well-tried didactic technique.

8 Commentary

Proclus presented his ideas on the *Elements* in a commentary, signaling the canonical status of the work. Proclus' commentary may have been based on lectures. As Thomas Heath noted, Proclus refers to "hearers",[79] and there is evidence that other commentaries were read out to students by teachers.[80] Proclus does indicate that he intends his audience to be students.[81] This is particularly interesting, because some modern authors have described the *Elements* as a 'textbook'.[82]

As part of the developing literary culture of the 'book', the didactic and scholarly traditions produced a variety of handbooks, epitomes, and commentaries; the works of Aristotle and mathematical texts (such as Nicomachus of Gerasa's) were often the topic of such treatments. Detailed scholarly exegesis of the Homeric poems was underway by the third century BC, and eventually philosophical and mathematical texts (as well as medical works) were also the focus of some very careful attention. While commentaries on various types of texts were important from the third century BC, the commentary was a particularly significant genre for scientific and mathematical writing in the later period. Theon of Alexandria (fl. 364 AD), apparently working with several collaborators, including his daughter Hypatia (d. 415 AD), prepared commentaries on a number of works, including Ptolemy's *Almagest*.[83]

78 Berryman 2009, 106; Hett 1936, viii; see also Coxhead 2012.

79 Proclus, *In Eucl*. 210.19, cf. 375.9; Heath 1921b, 532. However, because the vocabulary for 'listening to' and being a 'student', 'disciple', or 'follower' is related, it is not clear whether or not Proclus meant 'hearers' or, more generally, 'students'; see Taub 2008a, 14.

80 See Taub 2008a, 29.

81 See, for example, Proclus, *In Eucl*. 81–84; 210. See also Morrow 1970/1992, xxiv.

82 For example, Boyer 1968, 111.

83 Bernard 2008a, 423 f. Bernard 2008b, 793–795, includes a review of the attributions of commentaries and editions to Hypatia and Theon. Bernard 2008b, 794 (citing Theon's commentary on the *Almagest* 319.6–10, Rome edition) notes that Theon saw his own "interpretative stance [as a commentator] as the continuation of Ptolemy's own work as a commentator of the ancients, and urged the most able of his companions to go the same way".

Typically, a passage from the ancient source is quoted, and then a comment appended, which may be of any length, from one sentence to the equivalent of a number of pages. Additionally, the commentator may refer to other works, by the author of the target text, or other writers. Some of the commentators offer insights into issues concerning the understanding of the nature of mathematics and the work of mathematicians, issues alluded to at the beginning of the discussion here. In his commentary on Aristotle's *Physics* (193b23), Simplicius (sixth century AD) comments on the following passage (only briefly quoted here): "We must next consider in what way the mathematician differs from the physicist". Simplicius notes that Aristotle "quite justifiably wants to show the difference between the physicists and the mathematicians, since they appear to concern themselves with the same subjects".[84] Even as commentaries encouraged a close engagement with particular texts, they often served as vehicles for the presentation of the commentator's own ideas. This is the case in Proclus' *Commentary* on the *Elements*, in which, as head of the school of philosophy of Athens, the 'Academy',[85] he is concerned to a great extent with philosophical issues. Commentaries often functioned within teaching contexts, in which lectures and discussion took place; in his biography of his teacher Plotinus, Porphyry reports that "in our gatherings he would have the commentaries read out to him".[86]

With his students in mind,[87] Proclus saw part of his task as being to explicate the text of the *Elements*; to some extent, his treatment coincides with what we might expect from a literary or textual critic; his commentary may itself be a compilation.[88] In addition to considering the structures of the proposition (as detailed above), Proclus considers Euclid's mathematical writings, and his work specifically in composing the *Elements*.[89]

Ian Mueller, in a 1992 foreword to Morrow's translation of Proclus' commentary, helpfully contextualized Proclus' work as a Neoplatonist, as a teacher of philosophy and as a philosopher interpreting a mathematical text.[90] Netz, in a 1999

84 Simplicius, *In Arist. Phys. II* 290.1–5 (Fleet 1997, 45), slightly amended. Instead of 'physicist', Fleet translates "natural scientist", Hardie & Gaye in their translation of Aristotle's *Physics*, 193b23, use the phrase "student of nature". In the passage quoted, Simplicius discusses the passage at length, making references to both Aristotle's *On the Heavens* and Plato's *Timaeus*.

85 But as Mueller 1992, x–xi, notes, not the Academy of Plato.

86 Quoted by Grafton & Williams 2006, 34, citing Porphyry, *Vita Plot*. 14.10–14; see also Sluiter 2000, 191, on commentaries and oral teaching. On commentaries more generally see Most 1999 and Gibson & Kraus 2002.

87 Mueller 1992, ix and xxx–xxxi.

88 Heath 1921b, 534.

89 Proclus, *In Eucl*. 68–70.

90 Mueller 1992. But it is important to remember that many of the authors considered here were not, and should not be, considered to be simply 'mathematicians' or 'philosophers'. Although they may have had primary areas of interest in mathematics or philosophy, they often had views on a range of intellectual areas; See, for example, Tybjerg 2005 on Hero's philosophical views. Differences in the classification of knowledge (discussed briefly above) are also relevant here.

study of Proclus' description of the proposition, suggested that Proclus had himself devised the scheme and (possibly) the terminology to describe Euclidean propositions, and that he had done so as part of his project to produce a commentary on the *Elements*;[91] he has suggested that even if it was not devised by Proclus himself, it was probably not done by a mathematician, but by a philosopher producing a commentary on Euclid, and developing his own terminology (based to some extent on terms used by earlier philosophers).[92] Netz points out that "the scheme serves as the springboard for an extensive discussion of the philosophy of mathematics, in the commentary to the first proposition of the *Elements*", suggesting that "for Proclus himself, the scheme functions as a way of identifying the philosophical issues arising from mathematics".[93] Proclus composed his commentary within the context of his Neoplatonist Academy, as part of his teaching program. As a teacher of philosophy, Proclus used philosophical terminology and approaches to illuminate his reading of the *Elements*; Netz has argued that the scheme described by Proclus is particularly illuminating from a philosophical, rather than a mathematical perspective. In fact, some readers of Proclus' *Commentary* have regarded it primarily as a work of philosophy, even though the target text of his commentary was the most canonical of Greek mathematical works.[94]

9 Letter (Greek ἐπιστολή, epistolē)

Letters were important for communication generally in the Greco-Roman world and, as a genre, have particular import for certain communities (for example, the early Christians). Ancient Greek mathematicians also communicated via letters, and a number survive (although the genuineness of some has been questioned). Eratosthenes lived in Alexandria, where he was Librarian and royal tutor to Ptolemy's son Philopator, and was the recipient of letters from Archimedes, living in Syracuse. Archimedes corresponded with a number of individuals interested in mathematics; he apparently often sent out enunciations without proofs, that is, puzzles in advance of the works themselves.[95]

A very rich and, from the standpoint of genres, intriguing text is the *Letter* to Ptolemy III (Euergetes), attributed to Eratosthenes (*ca* 285–194 BC) and preserved

91 Netz 1999b, 302.
92 Netz 1999b, 302–303.
93 Netz 1999b, 302–303.
94 See, for example, Mueller 1992.
95 Netz 2004, 13. Interestingly, the *Oxford English Dictionary* lists as the first (but now obsolete) definition of "proposition": "Something proposed for discussion or solution; a problem, a riddle; a parable".

in Eutocius' (480–540 AD) *Commentary* on Archimedes' *Sphere and Cylinder*.[96] Here, within a commentary, we find a letter, which contains a proof, as well as poetry (a quotation near the beginning of the text and also an epigram, ascribed to Eratosthenes, which closes it).

The geometrical proof in this letter has the same basic format as we might expect, with some variation. The author described two different methods of finding mean proportionals, one by geometrical demonstration, the other using an instrument whose construction he describes. The author then goes on to explain that both the geometrical demonstration and the instrumental solution have been placed on a votive monument, as an offering from Eratosthenes of Cyrene, together with an epigram, extolling his solutions to the problem.[97]

10 Poem

Poetry was among the genres in which mathematical problems were presented. In the *Letter to Ptolemy*, mentioned above, the author first offers the problem to be solved by quoting an unnamed tragedian:

> μικρόν γ' ἔλεξας βασιλικοῦ σηκὸν τάφου
> διπλάσιος ἔστω, τοῦ καλοῦ δὲ μὴ σφαλεὶς
> δίπλαζ' ἕκαστον κῶλον ἐν τάχει τάφου.
>
> You have mentioned a small precinct of the royal tomb;
> Let it be double, and, not losing this beauty,
> Quickly double each side of the tomb.[98]

The letter is also concluded or signed with a poem, an epigram, which serves as a poetic seal or *sphragis*. The solution to the problem is presented, and celebrated

96 For the text of the *Letter*, see Eratosthenes, "Letter to King Ptolemy", in: Eutocius, ed. J. L. Heiberg Archimedis Opera (2nd ed., 1915) 3.88–96; see also the translation by I. E. Drabkin (1948) in *A Source Book in Greek Science*, eds. M. R. Cohen & I. E. Drabkin (New York) 62–66; portions of the *Letter* are edited and translated by Ivor Thomas (1939) in *Selections Illustrating the History of Greek Mathematical Works* (Cambridge, MA) 1.257–261 and 291–297, but the text from Eutocius is not completely reproduced there. Wilamowitz 1894/1971 thought that the letter was a forgery, but Thomas 1939, 256, note a, suggested that "there is no reason to doubt the story it relates".
97 I have discussed this letter in detail in Taub 2008a. Historians generally agree that the quotation which purports to be from the monument is genuinely the work of Eratosthenes.
98 Transl. Netz 2002, 214, slightly amended (Eutocius 88. 8–10, ed. Heiberg = 64. 10–12, ed. Mugler). Wilamowitz 1894/1971, 53–54, argued that these lines could not be from any play by the great Athenian tragedians, and must have been the product of a minor poet; cf. Thomas 1939, 258, note a. The fragment is *Trag. Graec. Frag.* 2.166.

in the epigram,[99] which even those who doubt the authenticity of the entire text attribute with confidence to Eratosthenes, because of its elegance and beauty:

> If you plan, of a small cube, its double to fashion,
> Or—good sir—any solid to change to another
> In nature: it's yours. You can measure, as well:
> Be it byre, or corn-pit, or the space of a deep,
> Hollow well. As they run to converge, in between
> The two rulers—seize the means by their boundary-ends.
> Do not seek the impractical works of Archytas'
> Cylinders; nor the three conic-cutting Menaechmics;
> And not even that shape which is curved in the lines
> That Divine Eudoxus constructed.
> By these tablets, indeed, you may easily fashion—
> With a small base to start with—even thousands of means.
> O Ptolemy, happy! Father, as youthful as son:
> You have bestowed all that is dear to the Muses
> And to kings. In the future—O Zeus!—may you give him,
> From your hand, this, as well: a sceptre.
> May it all come to pass. And may he, who looks, say:
> "Eratosthenes, of Cyrene, set up this dedication."[100]

The quotation from the tragic poet and the epigram confer a degree of literary interest and distinction on the *Letter*, while presenting a story-problem and its solution, which is worked out in detail in the central portion of the text.

Erastothenes' nickname in antiquity was 'Beta', acknowledging his accomplishments in a number of fields, while suggesting that he was not the highest achiever in any. However, a mathematician of the greatest renown in antiquity, Archimedes, also chose poetry as a way to present a mathematical problem. His *Cattle Problem* was offered as a poem. The text of the *Cattle Problem* was discovered and edited in 1773 by G. E. Lessing, who, as Eratosthenes had been, was employed as a librarian; while his predecessor had been in Alexandria, Lessing was at the Herzog August Library in Wolfenbüttel.

The text opens in the following way: "A Problem [*problēma*] which Archimedes devised in epigrams, and which he communicated to students of such matters at Alexandria in a letter to Eratosthenes of Cyrene".[101] (This is not the only letter

99 Within the *Letter* as a whole, it is made clear that the solution exists in a number of different formats, including the written proof, as well as the instrument which is described by the author as his innovation.

100 Transl. Netz 2002, 214 (Eutocius, *In Archim. Sphaer. cyl.* 96. 10–27 Heiberg = 68. 17–69. II Mugler). Knorr 1989, 144f. has suggested that the *Letter* was dedicated to the fourth King Ptolemy (Philopator), Eratosthenes' tutee, perhaps on the occasion of the endowment of royal honors on the infant heir apparent, the fifth Ptolemy (Epiphanes); on this reading, the *Letter* would have been written late in Eratosthenes' career. See also Wilamowitz 1894/1971, 65–66 on ambiguities in the epigram.

101 Transl. Thomas 1941, 202, with slight emendation.

from Archimedes addressed to Eratosthenes that we have.[102]) The problem itself is presented as a poem in epigrammatic form, forty-four lines in the modern edition. While some scholars have questioned whether Archimedes was responsible for the poem, most assume that he was familiar with the problem.[103] Unlike Eratosthenes' epigram, which provides the *sphragis* as a supplement to the prose letter addressed to Ptolemy, in the *Cattle Problem* the mathematical problem is itself tightly woven into the poetic format. Interestingly, no ancient prose setting of the mathematical contents of the problem is known.[104]

Here is a portion of the English prose translation by Ivor Thomas:[105]

> If thou art diligent and wise, O stranger, compute the number of cattle of the Sun, who once upon a time grazed on the fields of the Thrinacian isle of Sicily, divided into four herds of different colours, one milk white, another a glossy black, the third yellow and the last dappled. In each herd were bulls, mighty in number according to these proportions … .

Following the listing of the relevant proportions for each herd, both bulls and cows, the reader is then promised:

> If thou canst accurately tell, O stranger, the number of cattle of the Sun, giving separately the number of well-fed bulls and again the number of females according to each colour, thou wouldst not be called unskilled or ignorant of numbers, but not yet shalt thou be numbered among the wise. But come, understand also all these conditions regarding the cows of the Sun. When the white bulls mingled their number with the black, they stood firm, equal in depth and breadth, and the plains of Thrinacia, stretching far in all ways, were filled with their multitude. Again, when the yellow and the dappled bulls were gathered into one herd they stood in such a manner that their number, beginning from one, grew slowly greater till it completed a triangular figure, there being no bulls of other colours in their midst nor none of them lacking. If thou art able, O stranger, to find out all these things and gather them together in your mind, giving all the relations, thou shalt depart crowned with glory and knowing that thou hast been adjudged perfect in this species of wisdom.[106]

The solution requires finding the number of bulls and cows of each of four colors, or to find 8 unknown quantities. The seemingly simple question belies the surprisingly difficult character of the problem. Lessing published an incorrect solution; an ambiguity in the text contributed to J. F. Wurm's solution of a simpler form. In 1880 A. Amthor discussed the complete problem, and partly solved it. Amthor did

102 See also "The Method of Archimedes' Treating of Mechanical Problems – to Eratosthenes" in Heath 1912, 1–51, discovered by Heiberg in 1906 and recently the subject of study by Netz & Noel 2007.
103 Cf. Heath 1921b, 23.
104 Netz 2009, 167 f. See also Krumbiegel & Amthor 1880 on the problem.
105 To my knowledge, there is no English verse translation (though several German versions exist).
106 Thomas 1941, 202–205.

not write out the solution, but provided the first four significant figures; many accounts of the solution are based on his paper.[107]

Why was this intriguing problem presented as a poem? Wilbur Knorr suggested that Eratosthenes composed the first part of the problem, and that the second part is Archimedes' response.[108] There are a number of features of the *Cattle Problem* which reinforce links between Archimedes and Eratosthenes. First of all, there is the allusion to Homer's *Odyssey* and the cattle of the Sun. In the very opening lines of the *Odyssey* (I 6–10), there is a reference to the cattle of Helios, foreshadowing the forbidden slaughter of these livestock by Odysseus' companions in book 12. The number of animals (seven herds of cattle, and of sheep, with fifty in each), and the place where they pasture, the island of Thrinacia, is specified in book 12 (lines 127 ff.), when the goddess Scylla speaks to Odysseus: "… you will reach the island Thrinacia, where are pastured the cattle and the fat sheep of the sun god, Helios, seven herds of oxen, and as many beautiful sheep flocks, and fifty to each herd". Eratosthenes' interest in Homer was well attested, as is his interest in number theory (through the use of his sieve (*koskinon*) for finding successive prime numbers.[109] The *Cattle Problem* locates Thrinacia in Sicily, the home of Archimedes.[110]

By triangulating himself between Eratosthenes (arguably one of the greatest intellectuals of his age) and Homer (revered as one of the greatest Greek poets), Archimedes (if he was the author of the *Cattle Problem*) has highlighted intellectual bonds amongst the three, via numbers and poetry. But Eratosthenes and Archimedes, or whoever the authors of these mathematical poems might have been, were not alone in their interest in composing mathematical problems in poetry. The *Greek Anthology* has forty-odd poems which are mathematical problems presented as epigrams; many of these were collected by Metrodorus (*ca* 500 AD) but would have been written much earlier.[111] The number of mathematical poems that survive suggest that such poetry was not simply the reserve of these two correspondents, sending each other challenging problems. The relationship of problem-poems to story-problems suggests that there might be other generic issues involved.

107 Amthor 1880, 156 ff. See Vardi 1998 for a history of modern solutions. Sadly, I was unable to locate a copy of D. H. Fowler, "Archimedes' Cattle Problem and the Pocket Calculating Machine" (1980 with additions in 1980, 1981, and a postscript 1986), Warwick.

108 Knorr 1986, 295.

109 See Nicomachus, *Ar.* I 13 for a description of the "sieve".

110 See also Strabo, *Geogr.* VI 2.1 and Thucydides VI 2.2 on Thrinacia. Netz 2009, 34 and 167 f., apparently assumes that Archimedes is the sole author of the *Cattle Problem*.

111 See Paton 1918, 25–107.

11 Other genres of discourse about mathematics

In addition to those already discussed, there are other genres which were used by ancient Greek authors writing about mathematics. While a detailed consideration of any of these is not possible here, it nevertheless is important to emphasize the range and diversity of genres in which mathematics was presented and discussed.

Proclus refers to the *Elements* as a *pragmateia* (at 83.1), a word which the *Greek-English Lexicon* of Liddell-Scott-Jones (LSJ) suggests might be translated as "treatise".[112] The word 'treatise' is a modern term; the English word 'treatise' refers to a written work dealing formally and systematically with a subject. (The word's origin is Middle English *tretis*, from Old French *traiter*, and the Latin *tractare* meaning 'handle' or 'treat'.) Van der Eijk has pointed out that the "treatise" is a "less well defined species of text" sometimes referred to by this modern term; its style is usually considered to be less elaborate and its formal structure does not fit in with categories of prose recognized in antiquity such as dialogue, letter, commentary, handbook (*tekhnē*) and introduction (*eisagōgē*).[113]

Other ancient works dealing with scientific subjects, similarly described as treatises, appear to have begun as lectures;[114] as was mentioned earlier, it has been suggested that Proclus' *Commentary* on the *Elements* was a series of lectures and, indeed, the format of the work is somewhat different from what we encounter in some other commentaries, particularly with the two prologues which precede the work. It is worth noting too that Theon of Alexandria's recension of the *Elements* has as its title in some manuscripts "Lectures".[115] The boundaries between certain genres may well have been blurred because lectures, written down as notes, were edited for publication.

Another example of a written work which may have first been a series of lectures is Cleomedes' (*ca* 200 AD) text known as *The Heavens*. Cleomedes appears to have been a professional teacher; that *The Heavens* served a pedagogical purpose is indicated by the use of elementary argumentation and the frequent explication of terminology. At several points Cleomedes' language – which refers to 'lecture courses' (*skholai*) – suggests that the work probably had its origin as a series of lectures.[116]

112 Indeed, Morrow 1970, 68, translates *pragmateia* here as "treatise". See Liddell-Scott-Jones 1968, 1457.

113 Van der Eijk 1997, 89. The Greek term *pragmateiai* may also describe what we regard as 'treatises'; see Dirlmeier 1962, 9–11.

114 On the possible relationship between lectures and treatises, particularly relating to work of Aristotle, see Taub 2008a, 18–22.

115 On the editions of Euclid by Theon of Alexandria, see Heath 1921a, 360. I have discussed the oral character of certain genres, particularly lectures and poetry, in Taub 2008a, particularly 13–18.

116 Cf. Cleomedes, *Cael*. II 2.7 and II 7.12; see the translation by Todd & Bowen 2004, 127 and 165.

The publication of lectures as written works, in various forms including commentaries, is only one indication that there was an active market for pedagogical works in the Greco-Roman world. Certain works were intended to serve as introductions (*eisagōgai*) or teaching texts; several of these survive, including Nicomachus of Gerasa's (between AD 50 and 150) *Introduction to Arithmetic*, an elementary text on mathematics, and Geminus' (*ca* 50 AD) *Introduction to the Phenomena*),[117] concerned with astronomy. Both works begin with definitions, and it is relatively easy to imagine that they would have supplemented lectures; students may well have appreciated a written text to consult before and after the oral presentation.

Along with this work on arithmetic, Nicomachus produced an *Introduction to Harmonics*, an *Introduction to Geometry* (which has not survived), and possibly an *Introduction to Astronomy*. His *Introduction to Arithmetic* was used as a teaching text throughout later antiquity and into the Middle Ages (in a Latin paraphrase produced by Boethius (*ca* 480–*ca* 525 AD); a number of commentators, including Iamblichus (*ca* 245–*ca* 325 AD), Asclepius of Tralles (died *ca* 560/570 AD) and Philoponus (*ca* 490–570s AD), wrote about the work, indicating that it was the focus of further study itself.[118]

Iamblichus' *On the Pythagorean Life* was intended to serve as an introduction to a series of mathematical works. In this text, Iamblichus presents mathematics as a way of life and offers a narrative of the life of Pythagoras. Other lives (*bioi*) of Pythagoras were presented in the third century, notably by Diogenes Laertius and Porphyry, for whom the life (*bios*) of Pythagoras was only one amongst several accounts of lives of important figures they offered, within a larger work. Asper has considered the importance of narrative accounts given by Greek mathematical and medical writers; the account of a life is a particular type of narrative which may be the proto-genre of the scientific biography.[119] Certainly, like the proposition, it is a genre which often has a particularly mathematical stamp, and which was far-reaching in its impact, well beyond the boundaries of the ancient Greek world.

12 Then, and now

My aim in thinking about genres of discourse on mathematics is, in part, to try to understand the place of these texts in wider Greek (and Roman) culture. However, an interest in the genres of mathematical discourse is not restricted to the ancient period. In fact, there has in recent years been a surge of attention amongst mathe-

117 The dates for Geminus are not agreed; see Jones 1999, Bowen & Todd 2008, Taub 2003, 25.
118 See Toomer 1996a for publication details for these commentaries on Nicomachus. More generally, see Mansfeld 1998, 1–5.
119 See Asper, in this volume 421; see also Taub 2007.

matics teachers and pedagogical theorists to issues related to genre, authorial voice and other stylistic features in mathematical teaching texts.[120]

Two specialists in mathematics education, David Pimm and David Wagner, asked the following questions in 2003: “What *kinds* of mathematics are there? And what are possible bases for distinction or grouping, what are some salient features that could be stressed or ignored?” They suggested that there are several possible ways to address such questions: “One way, important both to libraries and to *Mathematical Reviews*, is by means of the traditional yet still-evolving categories such as “geometry”, “algebra”, “calculus”, “analysis”, and “number theory” – though these can generate turbulence at the boundaries, as well as increasingly requiring hybrids: algebraic geometry, topological algebra, analytic number theory, geometric topology, and so on”. Another possible way of “cutting up mathematics”, they suggested, “is to agree it is primarily written and then try to find bases (whether of form or function) for distinguishing and grouping types of writing into different kinds”. Pimm and Wagner then noted that “the question subsequently arises as to whether any observable differences are purely superficial or are in some way necessary, produced in response to demands of the situation: does form always have to follow function?”. They proposed that “an initial list might include the textbook, the published journal article, the written expository lecture, the letter (or increasingly e-mail message), the popular account or the encyclopedia entry, where each is also influenced by other non-mathematical examples of the ‘same’ form”.[121]

I began my consideration of the variety of genres of Greek mathematical writing by noting that, in common with modern mathematicians, ancient authors had many textual formats and modes of discourse available for communication. Then, as now, the genres used to communicate about mathematics mattered, and provide windows through which we see the interaction between technical literature, its authors and readers, and broader culture.

120 See, for example, Gerofsky 1999; Morgan 1998; Pimm & Wagner 2003.

121 Pimm & Wagner 2003, 159 f.

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