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Essay Review

# Satisfaction, subversion, and the reluctant reader: some thoughts on writting accessible history of ancient mathematics

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**Ancient mathematics**
Serafina Cuomo; Routledge, London and New York, 2001, pp. xii+290, Price £50.00 hardback, ISBN 0-415-16495-8, £16.99 paperback.

There are, as I see it, two target audiences for generalist books on the history of mathematics: historians and mathematicians. Both have particular sets of expectations and prejudices about the history of mathematics that may discourage them from even picking up such a book in the first place. (Historians of mathematics, I assume, are a relatively unproblematic category of readership.) Members of either group may intuitively feel that history of mathematics is something of an oxymoron: no more than a chronology of names of Great Mathematicians, each associated with one or more theorems or axioms, and perhaps the occasional colourful story (Archimedes and his bath; Galois and his duel) with about as much truth content as Grimm's fairy tales. Further, neither group tends to be particularly well grounded in the factual basis and methodology of the other's discipline. For most people calculating the area of a right-angled triangle or recalling the date of the Battle of Hastings are at or beyond the outer limits of general knowledge. How then to persuade these two diverse audiences to pick up a history of mathematics book in the first place, and satisfy their expectations enough to keep them reading it? At the same time it is probably desirable to subvert those expectations sufficiently that they find themselves engaged in, learning about, and enjoying both mathematics and history as they read, without skipping all the boring background or the incomprehensibly technical passages. Yet again, there is one's own authorial intention to satisfy: getting the balance

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right between the facile and the impenetrable, the historical context and the mathematical content, without patronising either readership or compromising one's own intellectual integrity or the sources themselves.

This is no small task for any author; but if your mathematics happens to belong to a long-dead society in an obsolete language your chances of success feel all the more remote. Even the current fashion for popular history of science has yet to spread beyond the boundaries of the modern world. Ancient historians, however engaged with the intellectual histories of the cultures they study, still tend to shy away from the more numerate aspects of their discipline. Mathematicians, on the other hand, may take one of three or more stances. Some feel that any pre-modern mathematics is trivial and irrelevant to modern mathematics. At the other extreme there are those who already feel ownership of some version of that past through often-repeated legends in the standard textbooks. They may either strongly resent changes to the received wisdom, or simply assume that because it all happened so long ago that there cannot possibly be anything substantively new to tell.

Of course, these sweeping statements are little more than caricatures, to which there are, and should be, many qualifications and exceptions. Whatever truth they might contain, they carry no imputation of blame on the part of the audience: almost all responsibility rests on the dry, internalist presentations of the subject that have always monopolised the field. But such have been my own, slightly dispirited feelings as I have pondered over the last year how to communicate Babylonian mathematics, my own particular specialism, outside my tiny circle of specialist peers. Is it possible to convey the richness, complexity, difficulty and sheer otherness that so engages me to an audience that for once I would like to be able to count not in dozens or even hundreds but perhaps into the thousands and beyond?

Thus Serafina Cuomo's *Ancient mathematics* landed on my desk at a particularly opportune moment. Here was an attractive, affordable work by a lively and likeable scholar of my own generation with a proven track record in innovative scholarship (e.g. Cuomo, 2000); how would it live up to, or help to recast, my ideals of the perfect history book — and how would it succeed in satisfying or subverting the desires of my imagined reluctant readers?

A short orientational introduction lays out the ground that Cuomo wishes to cover: 'not just . . . the advanced, high-brow practices, but also . . . 'lower' and more basic levels of mathematics, such as counting or measuring' (p. 1). Her express wish is to 'relate mathematical practices to their times and places, and to other cultural activities', which will entail 'look[ing] for mathematics in some strange places, including history and theology' and material culture (p. 2). The following eight chapters are, as she explains, laid out chronologically with a pair of chapters to each major period. The first of each pair, always subtitled 'The Evidence', aims to be a relatively 'full view of what we know of the mathematics of the period, what sources are available, a sketch of what is in the sources' (p. 2). The second, subtitled 'The Questions', raises 'a fraction of the questions that could be asked of the evidence in each case' without purporting to give definitive answers but rather to leave the big problems open for further debate, in class or in print (p. 3).

Here she has encapsulated two major and laudable innovations, which can be seen

in action throughout the book. First, she has spread her net beyond the traditional 'Big Guys' like Euclid and Archimedes to catch not only the 'Little People' of ancient mathematics but also philosophers, the 'Rest of the [non-mathematical] World' and 'Material Evidence' (e.g. p. 63 and *passim*). One might almost say that she is an ethnographer of ancient mathematics, interested as much in practitioners, their self-images, and their places in society, as in the mathematics itself (whether good, bad or indifferent). Her sources span the fifth century BCE to the sixth century CE, periodised into 'Early Greek' (Chapter 1, fifth–fourth centuries BCE, 35 pages), 'Hellenistic' (Chapter 3, third–second centuries BCE, 63 pages), 'Graeco-Roman' (Chapter 5, first century BCE – second century CE, 18 pages), and 'Late Ancient' (Chapter 7, third–sixth centuries CE, 20 pages), with almost half the weight of evidence coming from the Hellenistic period.

Second, the conscious problematisation of the narrative she tells enables the book to be read at two levels: the odd chapters taken together constitute a history of the traditional type (albeit with a very non-traditionally inclusive coverage), while the even chapters deal with some of the major problems in the historiography and social history of the mathematics just presented. Why, for instance, did mathematics feature so heavily in the political discourse of fifth-century Athens? Given the dearth of primary mathematical evidence from that period, how (if at all) can we disentangle the more or less reliable from the fantastical in origin narratives of later antiquity (Chapter 2, 23 pages)? Cuomo goes in search of 'the real Euclid' and examines the operation of mathematical communities in the Hellenistic world (Chapter 4, 18 pages). She vigorously tackles the old cliché, first enunciated by Cicero, that the Greeks treated mathematics as a purely intellectual art and held it in the highest esteem, while for the Romans it was no more than a utilitarian tool with manifold applications to the real world. This is Cuomo's particular forte. She first unpacks the Ciceronian tradition, then looks again at the self-descriptions of the mathematicians and mathematical practitioners themselves (Chapter 6, 49 pages). Finally, she tackles the relationship between numerology and Christianity and examines Late Antiquity's view of its mathematical past — which was by this time a thousand-year-old heritage (Chapter 8, 37 pages).

Her questions, then, are more historical than mathematical: the mathematics itself by and large remains unproblematised, unquestioned. And this, for me, is related to one of the more disappointing aspects of the book: that much of the mathematics itself is presented as an optional, unmediated extra. Longer and more difficult passages are separated from the main text by heavy horizontal rulings and we are gaily told that 'non-mathematically-inclined readers can just ignore them' (p. 2). But even the mathematically-minded reader might need some assistance in reading these very verbose texts, densely packed with lettered references to diagrams, which are so different from the terse, symbolic mathematical discourse of the modern world. We are left to struggle on alone, with only the aid of a short glossary (pp. 263–6) to help us.

How did this tradition of the disquisition upon a lettered diagram arise and develop? Cuomo's footnotes refer several times to Reviel Netz's masterly book *The shaping of deduction in Greek mathematics* (1999) but nowhere in the main body

of the work does she address the *mathematical* questions it raises; rather, she summarises its consequences for the social history of mathematicians in her section on the Hellenistic mathematical community (pp. 135–6). There is even less engagement with another provocative masterpiece of recent years, David Fowler's *The mathematics of Plato's academy* (2nd. ed., 1999). This too is about mathematical origins, challenging the received wisdom that the discovery of incommensurability provoked an intellectual crisis leading to the non-arithmetised tradition epitomised by Euclid's *Elements*. Fowler's work merits no mention in the footnotes to Chapters 1–4, where one might expect it; rather, his collected data on papyrological sources of the Graeco-Roman period is cited in Chapter 5 (note 3, p. 188).

Both Fowler and Netz are very engaged with the material culture of mathematical documents, but this concern is conspicuously absent from Cuomo's book. All of its meagre twelve illustrations are of *realia* such as sundials and surveying instruments: as already mentioned, she deals admirably and engagingly with the materiality of evidence not usually called upon in histories of mathematics, and which is almost without exception contemporaneous with the periods under review. (The only precedent, to my knowledge, is Dilke's engaging little *Mathematics and measurement* (1987).) But disappointingly there are no pictures of the core mathematical documents themselves, and their physical form is addressed only in the section on 'the real Euclid' (pp. 126–35). From this discussion, with careful reading, one can deduce that there are only three extant manuscripts of Euclid's *Elements* from before the tenth century CE, and that these are all tiny fragments of papyri; but nowhere is it explicitly stated that — with very exceptional exceptions like these — all the mathematics in Cuomo's book, whether by the Big Guys or the Little People, is extant *only* in nth-generation manuscript sources that are chronologically closer to our time than to their original date of composition. This may be obvious to any classical historian but for mathematicians, or even historians of later mathematical cultures, this fact has to be spelled out as clearly and prominently as possible. A few months ago a colleague of mine got her beginning history of maths students to watch a *Horizon* television programme on BBC2 about a recently rediscovered manuscript containing important works by Archimedes. When she asked her class who they thought had written the object under investigation, the artefact itself, the majority answered 'Archimedes'; very few had registered that it was in fact a tenth-century copy.

Even if we rarely have access to 'original' sources, whatever we mean by that, the distance from manuscript(s) to print edition is long and often tortuous. What do mathematical manuscripts look like? And how do papyri from Hellenistic Egypt differ in language, content, and presentation from later Arabic or Greek codices — or from the Teubner editions of a century ago which for all practical purposes serve as primary sources for many historians of ancient mathematics? Some answers to questions like this would have fitted comfortably with the historiographical sections on Late Antique presentations of early mathematics Cuomo gives in Chapters 2 and 8.

This in turn brings me to my last criticism of Cuomo's book (which in this respect belongs to a venerable tradition indeed): the privileged and unqualified use of the

word 'ancient' to refer to the Mediterranean world during the millennium after *c*.500 BCE. This is not a usage, I imagine, that most non-specialist readers would question or misunderstand, but it is deeply problematic. The intellectual community as a whole is, I hope, operating on a post-colonialist paradigm, which no longer unquestioningly assumes that history always begins in the Classical world. How, otherwise, to describe the mathematics of much older cultures across the Middle East and Asia, from Egypt to China? Mesopotamia (Iraq), for instance, had been producing significant quantities of non-utilitarian mathematics for over a millennium before the dawn of Greek mathematics; its arithmetical and metrological roots predate even the development of literacy in the fourth millennium bce (see most recently Høyrup, 2002; Høyrup and Damerow, 2001). Such Eurocentric presuppositions lead Cuomo (following the standard Classicists' line) to date the birth of grammar to the Hellenistic period (p. 139) — already well attested in southern Iraq from the early second millennium bce onwards (Black, 1991) — and the invention of the catapult to 399 BCE (p. 63) although the Egyptian and Assyrian armies were using them at least 350 years earlier (see, for example, Schulman 1995: 298–9). But these are peripheral details, and I am thankful at least that the three clear maps (pp. ix–xi) make it clear from the beginning that the book encompasses the whole Mediterranean and not just Greece and Italy.

Cuomo is wise, however, to steer clear of the standard teleological treatment of Egypt and Mesopotamia as precursors or harbingers of Classical civilisation (p. 2; see Bahrani, 1998; Fowler, 1999b). The relationship is much more interesting and complicated than that, but can hardly be properly evaluated yet. Prerequisites include a fuller publication of the mathematics from Persian, Hellenistic, and Parthian (Roman) Mesopotamia, only a small fraction of which is currently in the public domain, and a thorough re-evaluation of the Demotic mathematical papyri from Egypt, now in preparation by Annette Imhausen. Nevertheless, there is already a plethora of well-known examples of Graeco-Roman and Late Antique mathematics which stand in conspicuous contrast to the continuing Euclidean tradition yet are famously reminiscent of Egypto-Mesopotamian mathematical styles, based on word problems with pseudo-real-world settings and specific numerical examples. Cuomo discusses Heron of Alexandria (Egypt, first century CE, pp. 161–8), Nichomachus of Gerasa (Jordan, first–second second century CE, pp. 181–3), and Diophantus (first or third century CE, pp. 218–23), without, however, making reference to the clear stylistic parallel with the Demotic mathematical papyrus from Hellenistic Hermopolis (Egypt, third century BCE) that she quotes on pp. 70–71.

Cuomo also quotes intriguing evidence from some Graeco-Roman Little People, previously unknown to me. An anonymous Egyptian papyrus writer in the second century CE poses a problem about finding the length of a field of known width and area (p. 146) — a problem type attested in southern Mesopotamia from at least the twenty-third century BCE (see most recently Robson & Foster forthcoming). And one Columella, a Roman of the mid-first century CE, quantifies the needs of agricultural year (p. 175) in a style highly reminiscent of the Sumerian school composition *The Farmer's Instructions* from early second millennium Mesopotamia (Civil, 1994). There are suggestive parallels here with Alexander Jones' seminal study of papyri

from Hellenistic and Roman Egypt (1999) and related articles, which have revealed a heady mix of Babylonian, Egyptian and Greek traditions in post-Classical astrology and astronomy. When and how cross-cultural mathematical contacts were made remains to be seen, but it certainly appears to have been a very Middle Eastern phenomenon. I also idly wonder what part, if any, the Aramaic language played in that post-Alexandrian scientific diffusion.

But, as I have suggested, the time is not yet ripe for a full investigation into such problems. Meanwhile, I shall attempt some answers to my original question: how well does *Ancient mathematics* serve its potential readers, mathematical and historical? First, it must be said that this book is on many levels a resounding success. Cuomo breaks welcome new ground in presenting ancient mathematics as an essentially human activity, carried out in social settings ranging from the scholastic to the academic, from the military to the agricultural, from the political to the religious. For this refreshing insight alone — which will come as a revelation to those for whom Heath (1921) or Neugebauer (1952) is still the last word on Greek mathematics (there being no Roman counterpart to speak of in this view) — she deserves our heartfelt thanks. By this means, and by giving equal weight to the questions and to the sources, she brings mathematics firmly into the realm of history as other historians know it.

One half, then, of my putative reluctant readership should have their expectations satisfied. I doubt, though, whether many of them will find themselves sucked into reading the heavily fenced off mathematical extracts; I confess I skipped most of them too. There must be ways of weaving them into the narrative so that the intellectual content of the mathematics becomes as important to the story as the authors, practitioners, and communities that fostered it. Mathematically-minded readers, on the other hand, might well enjoy getting stuck into many of these extracts, even if they default into translating them into modern symbolism for want of guidance about how else to interpret them. Cuomo is to be fêted, though, for including not a single equation in the whole 290 pages.

Readers with little or no Classical history, sadly, will struggle with the very dense texture of the book. I have already expressed my disappointment that there is no discussion of the nature of the manuscript sources. Other types of background knowledge are taken for granted too. How long are 2 1/2 *schoenia* (p. 146) for instance, and who was Xenophon (p. 42)? Neither are in the index, the glossary covers only mathematical terminology, and there is not even a rudimentary chronological chart. It is generally a hard book to navigate, especially for readers used to heavily signposted and cross-referenced scientific works. Putting some more thought and effort into simple support tools would have markedly increased the book's accessibility to non-Classicists. (Routledge publishes a long and well-regarded list of ancient histories aimed at the undergraduate market; it would have been easy enough to point to one or more of those.)

Mathematics is about ideas, mathematics is about people and societies. Mathematics thus has a history worth telling and Cuomo succeeds admirably in communicating that message, thereby facilitating its long-overdue entry into the mainstream of ancient historical discourse. Even if she does not succeed equally in satisfying and

subverting the diametrically opposed wants of mathematically and historically minded readers, she comes far closer to that goal than any of her predecessors. *Ancient mathematics* sets new standards for communicating a notoriously unattractive subject to a hitherto unwilling audience — and if those standards are maintained and built on by her successors we will be fretting about reluctant readers no longer.

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